WAVEGUIDES

$n_s = 1{,}445$

$n_c = 3{,}47$

$\lambda_0 = 1550\,nm$

$n_{eff}^{TE} = 2{,}69 \qquad n_g^{TE} = 3{,}963$

1) Calculate $\beta(\omega)$ of the mode

$\lambda = \frac{c}{f} \qquad f_0 = 193{,}6\ THz \qquad \omega_0 = 2\pi f_0 = 1{,}216 \cdot 10^{15}\ \frac{rad}{s}$

$$\beta(\omega) = \frac{2\pi}{\lambda} n_{eff}(\lambda) = \frac{\omega}{c} n_{eff}(\omega)$$

$$\beta(\omega) = \beta_0 + \beta_1(\omega - \omega_0) + \frac{\beta_2}{2}(\omega - \omega_0)^2 + \dots \qquad \frac{rad}{m}$$

Neglect β_2

$$\beta_0 = \frac{\omega_0}{c} n_{eff}(\omega_0) \simeq 1{,}09 \cdot 10^7\ \frac{rad}{m}$$

$$\beta_1 = \left.\frac{\partial \beta}{\partial \omega}\right|_{\omega_0} = \frac{\partial}{\partial \omega}\left[\frac{\omega}{c} n_{eff}(\omega)\right]\Bigg|_{\omega_0} = \frac{n_g(\omega_0)}{c}$$

$$\beta_1 = -1{,}321 \cdot 10^{-8}\ \frac{s}{m}$$

$$\boxed{\beta(\omega) = 1{,}09 \cdot 10^7 + 1{,}321 \cdot 10^{-8}(\omega - \omega_0)}$$

2) Phase velocity at ω_0

$$v_p(\omega_0) = \frac{\omega_0}{\beta(\omega_0)} = \frac{\omega_0}{\beta_0} = \frac{c}{n_{eff}(\omega_0)} = 1{,}12 \cdot 10^8\ m/s$$

3) Group velocity

$$v_g = \frac{\partial \omega}{\partial \beta} = \frac{1}{\beta_1} = \frac{c}{n_g(\omega_0)} = 7{,}57 \cdot 10^7\ m/s$$

4) Assume $\beta_2 = 2{,}3 \cdot 10^{-21} \; s^2/m$

Delay between two frequencies with $\Delta\lambda = 1nm$ after 1mm propagation in the waveguide

$$\Delta\tau = \frac{L}{v_g(\omega_1)} - \frac{L}{v_g(\omega_2)} \simeq L \frac{\partial^2 \beta}{\partial \omega^2} \Delta\omega$$

$$\Delta\omega = 2\pi \frac{c}{\lambda^2} \Delta\lambda \simeq 7{,}85 \cdot 10^{11} \; rad/s$$

$$\Delta\tau \simeq 1{,}8 ps$$

MACH-ZEHNDER DESIGN

$n_{eff} = 1,46$ $\quad n_g = 1,51$ $\quad \Delta\lambda = 0,8\,nm$ $\quad \lambda_1 = 1550\,nm$

1) Find the unbalance

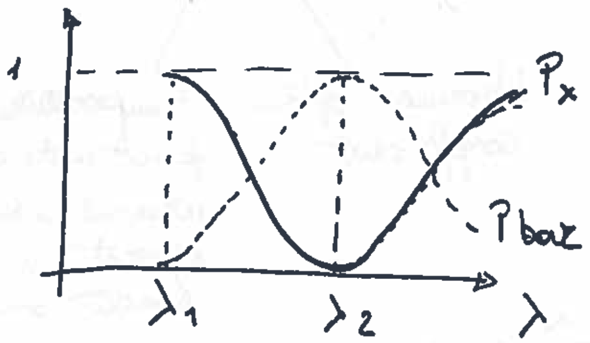
$$\Delta f = \frac{c\,\Delta\lambda}{\lambda_0^2} = 99,84\ GHz$$

$$FSR = \frac{c}{n_g \Delta L} \Rightarrow \Delta L = 994,93\,\mu m$$

To ensure $\lambda_1 = 1550\,nm$

$$P_x(\lambda_1) = 1 \Rightarrow \frac{\pi\, n_{eff}\, \Delta L}{\lambda_1} = N\pi \Rightarrow N = 937,76 \Rightarrow \boxed{N = 937}$$

$$\boxed{\Delta L = 994,76\,\mu m}$$



2) What if the two couplers are identical but not 50/50?

$K L_c \neq \frac{\pi}{4}$ $\quad$ (L_c : length of the couplers)

$$P_x = \cos^2\left(\frac{\Delta\varphi}{2}\right)\sin^2(2KL_c) = 0 \Rightarrow \cos^2\left(\frac{\Delta\varphi}{2}\right) = 0 \Rightarrow$$

$$\Rightarrow \frac{\Delta\varphi}{2} = (2N+1)\frac{\pi}{2} \Rightarrow \Delta\varphi = \pi(2N+1)$$

$\boxed{P_x = 0 \text{ is possible}} \Rightarrow P_{bar} = 1$ is possible

$P_{bar} = \cos^2\left(\frac{\Delta\varphi}{2}\right)\cos^2(2KL_c) + \sin^2\left(\frac{\Delta\varphi}{2}\right) = 0$ NOT POSSIBLE

$\boxed{P_{bar} = 0 \text{ IS NOT possible}}$

3) Tolerances

We want the cross-talk on the isolated port $x < -20\,dB$

Ideally $P_x(\lambda_2) = 0$ and $P_{bar}(\lambda_1) = 0$

$$\Delta\varphi = \frac{2\pi}{\lambda} L_{eff} =$$

$$= \frac{2\pi}{\lambda} \Delta L\, n_{eff}$$

Phase fluctuation : $\delta\varphi$

a) Maximum temperature variation

$K_{TH} = 10^{-5}$: thermo-optic coefficient

$$\delta\varphi = \frac{2\pi}{\lambda}\Delta L\,\delta n_{eff} = \frac{2\pi}{\lambda}\Delta L\, K_{TH}\,\delta T$$

δT ← temperature variation

Remember that

$$\lim_{\vartheta \to 0} \frac{\sin(\vartheta)}{\vartheta} = 1 \quad (\text{Use Taylor expansion} \ldots)$$

$$\Rightarrow \sin\vartheta \simeq \vartheta \text{ if } \vartheta \to 0$$

First choice: $P_{bar}(\lambda 1 = 1.55\mu m) = 0$

$$P_{bar} = \sin^2\left(\frac{\Delta\varphi}{2} \pm \frac{\delta\varphi}{2}\right) < X = 10^{-2} \quad (-20\,dB)$$

↑ phase fluctuation due to temperature variation

At $\lambda = 1{,}55\mu m \Rightarrow \frac{\Delta\varphi}{2} = N\pi = 937\pi$ (from the design)

$$\sin^2\left(937\pi \pm \frac{\delta\varphi}{2}\right) = \sin^2\left(\frac{\delta\varphi}{2}\right) \underset{\delta\varphi \simeq 0}{\approx} \frac{\delta\varphi^2}{4} < X$$

$$\frac{\pi^2}{\lambda_1^2}(\Delta L\, \delta n_{eff})^2 < X \Rightarrow \boxed{\Delta L\, \delta n_{eff} < \frac{\lambda_1}{\pi}\sqrt{X}} = 49\ \text{nm}$$

$$\Rightarrow \boxed{\delta T < 5°C}$$

Second possibility : $P_{cross}(\lambda_2 = 1550{,}8\ \text{nm}) = 0$

$$P_{cross} = \cos^2\left(\frac{\Delta\varphi}{2} \pm \frac{\delta\varphi}{2}\right) < 10^{-2}$$

at $\lambda_2 = 1550{,}8\ \text{nm} \Rightarrow \frac{\Delta\varphi}{2} = 1873\frac{\pi}{2}$

$$\cos^2\left(1873\frac{\pi}{2} \pm \frac{\delta\varphi}{2}\right) = \sin^2\left(\frac{\delta\varphi}{2}\right) < 10^{-2}$$

$$\Rightarrow \delta T < 5°\ C$$

b) Maximum birefringence

$$\delta\varphi = \frac{2\pi}{\lambda}\Delta L \cdot B$$

birefringence : $B = n_{eff}^{TE} - n_{eff}^{TM}$

$$X = 10^{-2} \Rightarrow \Delta L\, \delta n_{eff} < \frac{\lambda_1}{\pi}\sqrt{X} = 49\,\text{nm} \Rightarrow \boxed{B < 10^{-5}}$$